Physics 604
Mid-term Exam

1) a) Consider the 2-D Laplace equation in polar coordinates $\left(r=\sqrt{x^2+y^2}, \theta=\tan^{-1} y/x\right)$.

a) (10 pts.) Show that the functions $\psi = r^{\pm\nu}\cos(\nu\theta)$ have vanishing 2-D Laplacian.

$$\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial \psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2} = 0.$$

b) (5 pts.) What is a potential function for a uniform electric field $\vec{E} = E_0 \hat{x}$, expressed in the standard polar coordinates?

c) (5 pts.) What is a potential function for a uniform electric field $\vec{E} = E_0 \hat{y}$ expressed in polar coordinates?

d) (10 pts.) A long grounded conducting circular cylinder of radius a is placed so that the symmetry axis is aligned with the z -axis normal to the *xy* plane. At large distances $r \to \infty$ the electric field becomes uniform in the *x*-direction. Find an expression for the total potential for $r > a$ satisfying the relevant boundary conditions. (Hint: Do NOT look for images. Look something you can add to the potential of the uniform field to make the potential at $r = a$ vanish.)

e) (10 pts.) For the situation described in part d), what is the surface charge density on the cylinder as a function of θ?

2) a) (10 pts.) Represent the function shown in Figure 1 as a Fourier series in the interval $[-\pi, \pi]$.

b) (10 pts.) What is

$$\sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} \frac{\sin(n \times 0.5)}{n} = -\sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} \frac{\sin(-n \times 0.5)}{n} ?$$

Aside from the function value $x = 0.5$, at what other values of x does the summation achieve the same value?

3) Consider the azimuthally symmetrical electric field described by the potential $\Phi(r,\theta) = A_l \left(a^{l+1}/r^{l+1}\right) P_l(\cos\theta)$

a) (10 pts.) What is the total electrostatic field energy contained in the radial electric field E_r for $r > a$?

b) (10 pts.) What is the total electrostatic field energy contained in the polar electric field E_θ for $r > a$? (Hint: After computing E_θ and squaring, use the differential equation for P_l to allow a straightforward evaluation of the $\cos\theta$ integral)

c) (10 pts.) If $\Phi(r,\theta) = A_l\left(a^{l+1}/r^{l+1}\right)P_l(\cos\theta) + A_{l'}\left(a^{l'+1}/r^{l'+1}\right)P_{l'}(\cos\theta)$, what is the total field energy for $r > a$ in terms of $l, l', A_l, A_{l'}$

d) (10 pts.) On the z -axis the potential is $\Phi_{z-axis}(z) = \frac{2\ \mathrm{V\ m^2}}{z^2} + \frac{4\ \mathrm{V\ m^4}}{z^4}$ for $z > a$ in an azimuthally symmetric problem. What is $\Phi(r,\theta)$ and the energy stored in the field for $r > a$?

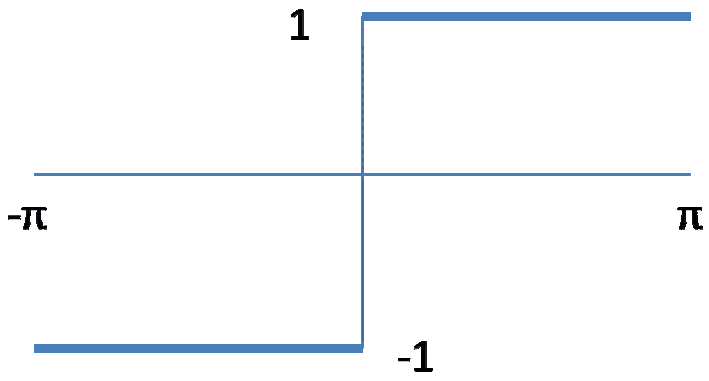

Figure 1